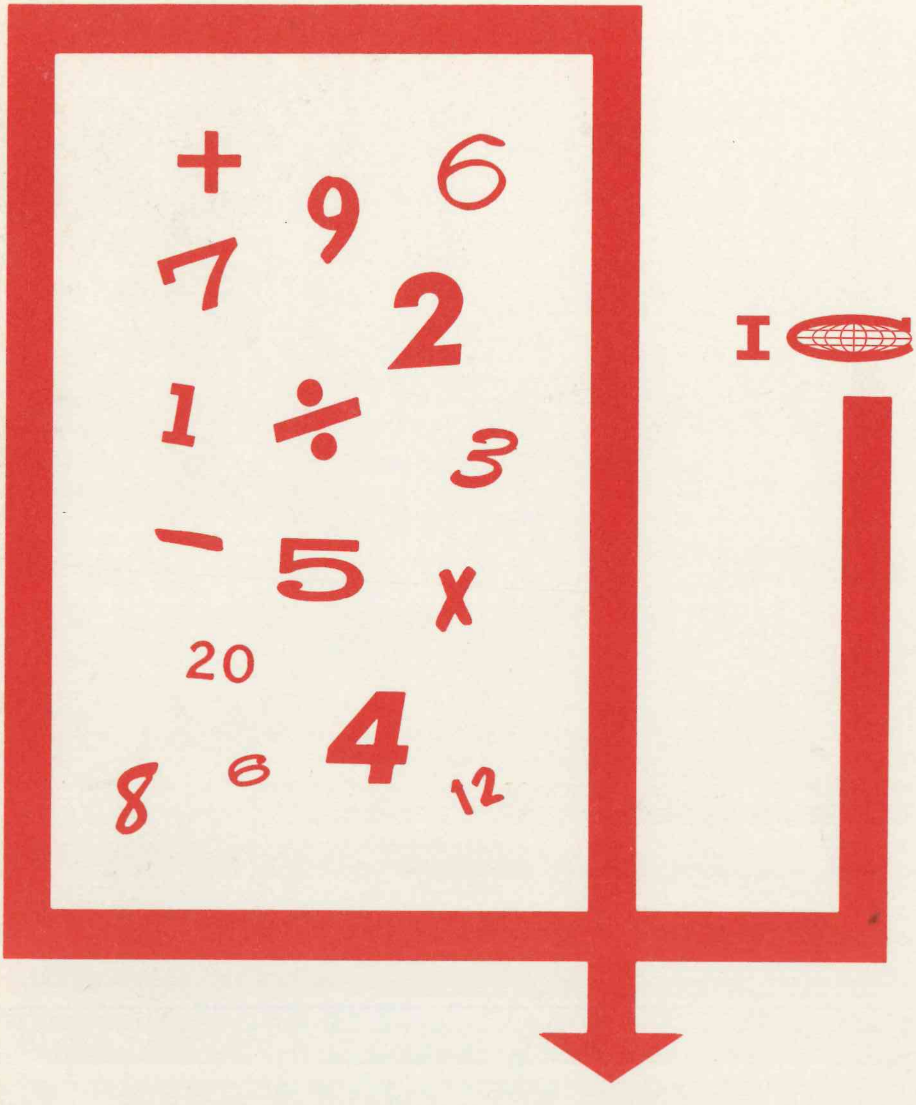

ICS®

# Practical Arithmetic

## Ratio, Proportion, and Per Cent

By

W. M. SPANGLER, M.A.

6750F
Part 6
Edition 1

# Practical Arithmetic

## PART 6

# Ratio, Proportion, and Per Cent

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## *What This Text Covers . . .*

In this, the final text in *Practical Arithmetic,* you have reached an important milestone in your studies. You have learned many things, but above all we hope you have learned how to apply what you have learned and especially how to use that very rare element called "common sense." In this text you will find more applications of this ability.

1. COMPARING NUMBERS . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 1 to 4

   *You will be amazed at the many comparisons we make almost unconsciously. Many of these comparisons are not very accurate, and many times they are not based on facts. In this section we shall learn how we can compare quantities or numbers representing them so that our comparisons are valid.*

2. RATIO . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 5 to 10

   *Here you will learn about ratios. First you'll learn what a ratio is; then you'll learn how to read and write it; and finally you'll learn how to find its value.*

3. PROPORTION . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 11 to 18

   *A definition of proportion is given early in this section. This is followed by instructions on how to set up a proportion for certain given conditions. This section is very important, so be sure to learn it well.*

4. APPLICATIONS OF PROPORTION . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 19 to 24

   *In this section you will learn how to solve problems by using ratios to form a proportion for the problem. In practical work there are many situations that require a knowledge of ratio and proportion.*

5. THE UNIT METHOD . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 25 to 28

   *For those who find it difficult to solve problems by ratio and proportion, we give another method, called the unit method. The reasoning upon which this method is based can be very effective.*

6. PER CENT . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Pages 29 to 36

   *Here is another way to compare numbers, but it is so important in other respects that we often fail to associate it with methods of comparison. Many of the various types of problems involving per cent are explained here.*

# Practical Arithmetic

## PART 6

# Ratio, Proportion, and Per Cent

## *Comparing Numbers*

### Comparisons

1. Making comparisons is a very common practice today. The buyer of a car compares the prices of the cars he is interested in. The housewife makes a comparison of the prices in different stores before she places her order. A great deal of what you read in the newspapers and magazines is based upon either a direct comparison (Brand A is better than Brand B) or an implied comparison (This is the best toothpaste you can buy). How often have you heard expressions such as the following?

I like this dress better than the blue one.
Your car is larger than my car.
Jim is a better ballplayer than Joe is.
This model is the more economical one.
It takes less fuel to operate this burner.
This winter was more severe than last winter.

In fact, we could give many other examples of statements that either make a direct comparison or imply that a comparison has already been made.

### Comparison of Numbers

2. In this text we are going to learn how to make comparisons and how to express them. Of course, our methods will have to be based upon scientific observations and facts and our results will have to be expressed in precise terms. Opinion, or matters that involve opinion, will not enter into our considerations. Examples of the kind of comparisons that we will be interested in are found almost everywhere. Just keep your eyes and ears open and be alert. Here are some of the things you will see and hear: Newspaper articles showing the relation between the cost of government now and the cost in some preceding year. Figures comparing the number of births in a community with the number of deaths in the same period. A comparison between the cost of living at the present time with the cost ten years ago.

Another example of comparing numbers is found in government health reports. One of these may state that for each 1000 inhabitants in a certain city 25 persons suffered from influenza, 1.24 from heart failure, and so on. Reports given in this form are much more instructive than those which simply state that in the same city 1250 persons suffered from influenza, 62 from heart failure, and so on. By means of the first report it is possible to make comparisons with the health situation in other cities; but with reports of the second kind, accurate comparisons are impossible unless the numbers of inhabitants are known. The important point in each of these instances is that two numbers are required in order to make a comparison. Now we shall learn how to make a comparison of the two numbers.

### Comparing Numbers by Subtraction

3. Very often we compare two quantities simply by saying that one quantity is greater than the other or that one quantity is less than the other. Here are a few examples of such comparisons: 1) In a football game the number of points made by the home team was greater than the number made by the visiting team. 2) In 1960 the number of people living in New York City, or the population of New York City, was greater than the population of Chicago. 3) The distance from New York City to Chicago is less than the distance from New York City to Miami Beach. 4) The cost of one machine is less than the cost of another machine.

In these comparisons no information is given to show *by how much* one quantity is greater or less than the other quantity of the pair. It is often desirable to compare two quantities more definitely. One way to do this is to subtract one quantity from the other and give the difference. For example, suppose that the home team won the football game by the score of 12 to 7. You could then say that the number of points made by the home team was 5 more than the number made by the visiting team. Or you could compare the same two quantities by saying that the number of points made by the visiting team was 5 less than the number made by the home team.

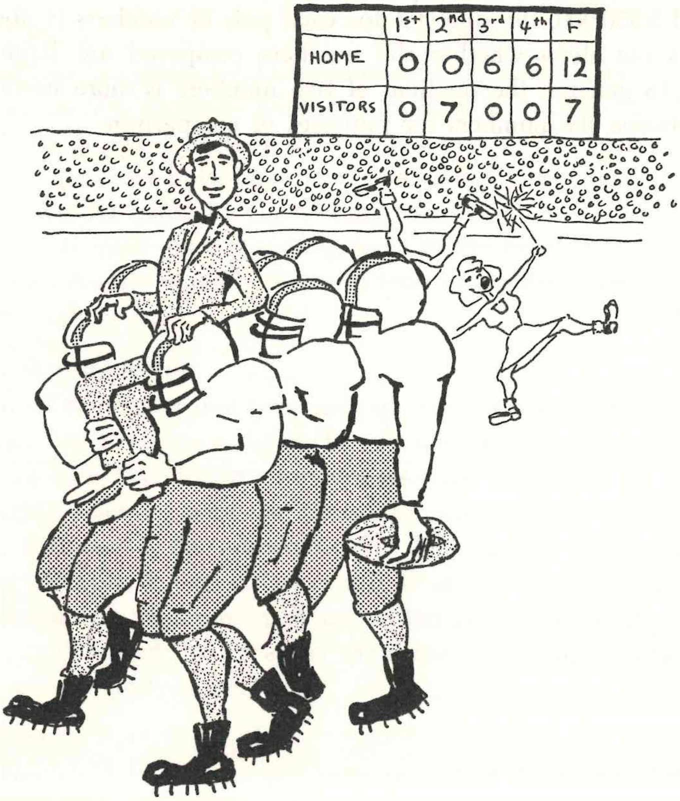
In 1960 the population of New York City was about 7,780,000 and the population of Chicago was about 3,550,000. You could say that, in 1960, the population of New York City was about 4,230,000 greater than the population of Chicago or that the population of Chicago was 4,230,000 less than the population of New York City.

If you know the distances from New York City to Chicago and from New York City to Miami Beach, you can find the difference between the distances and give it when you compare them. Likewise, if you know the cost of two machines, you can give the difference between the costs in a statement comparing the costs.

You should not need any further explanation of how you compare two quantities by giving the difference between them. You simply say that one quantity is so much *greater than,* or so much *less than,* the other quantity.

### Comparing Quantities by Division

4. Another way to compare two quantities is to divide one quantity by the other and give the quotient. Consider the four comparisons given in the pre-



ceding article. The first one is based on the fact that the home team won the game by a score of 12 to 7. Since 12 divided by 7 is nearly 2, you can say that the number of points made by the home team was nearly twice the number of points made by the visiting team. Or, since the value of the fraction $\frac{7}{12}$ is just a little more than $\frac{1}{2}$, you can say that the number of points made by the visiting team was about half the number of points made by the home team.

The second comparison is between 7,780,000 and 3,550,000. Since the quotient obtained by dividing 7,780,000 by 3,550,000 is more than 2, you can say that, in 1960, the population of New York City was more than twice the population of Chicago. Or you can say that the population of Chicago was less than half of the population of New York City.

In a similar way you can compare the distances from New York City to Chicago and from New York City to Miami Beach by dividing one distance by the other. Or you can compare the known costs of two machines by dividing the cost of one by the cost of the other.

You have probably noticed that the quotient of 12 divided by 7 is a little less than 2 and that the quotient of 7,780,000 divided by 3,550,000 is a little more than 2. Even though the numbers 12 and 7 are much smaller than the numbers 7,780,000 and 3,550,000, the quotient for each pair of numbers is about 2. The quotient does not show whether the numbers compared are large or small. Nevertheless, in general, the quotient of two numbers is more useful than the difference between the numbers for purposes of comparison.

## *Ratio*

### Meaning of Ratio

5. When two quantities are to be compared by dividing one quantity by the other, it is not always necessary to do the division. Very often the division is just *indicated.* This indicated division of one quantity by another is called a *ratio.* In our football game example the ratio of the number of points made by the home team to the number made by the visiting team is 12 to 7, or the ratio of the number of points made by the visiting team to the number made by the home team is 7 to 12. A ratio is really a special kind of quotient. And this is what we are going to study next.

### Methods of Writing Ratios

6. You must remember that two, and only two, numbers are compared by the use of a ratio. You are going to learn the two methods of writing the ratio of one number to another that are in common use. One method is to write the two numbers with a colon (:) between them. The other method is to write the two numbers so that one is the numerator and the other is the denominator of a fraction. For example, if two distances are 30 in. (inches) and 12 in., the ratio of the first distance to the second may be written as 30:12 or as $\frac{30}{12}$. When reading a ratio, you say the word "to" *for either the colon or the fraction line.* Thus either 30:12 or $\frac{30}{12}$ is read "thirty to twelve."

### Terms of Ratio

7. The two numbers written as a ratio are called the *terms* of the ratio. For example, in 30:12, or $\frac{30}{12}$, the numbers 30 and 12 are the terms of the ratio. Furthermore, since 30 appears first, it is called the *first term*; and 12, which appears second, is called the *second term.* The first term of a ratio may be either greater or less than the second term.

The terms of a ratio do not need to be whole numbers. Either term may be a proper fraction, a decimal, a mixed decimal, or a mixed number. Examples are $1:2\frac{1}{2}$, $\frac{3}{4}:\frac{5}{12}$, 6.38:4.17, and 0.25:4. Each of these ratios can be written as a fraction

$$\frac{1}{2\frac{1}{2}} \quad \frac{\frac{3}{4}}{\frac{5}{12}} \quad \frac{6.38}{4.17} \quad \text{and} \quad \frac{0.25}{4}$$

### Check Your Learning

Here is the first group of "check your learning" statements. As in Parts 1 to 5 of *Practical Arithmetic,* the statements are numbered consecutively throughout the text. You'll again find the correct answer slightly below and to the right of each statement. Cover our answer with a piece of paper and complete the statement by writing the best answer you can in the space provided. Then compare your answer with ours. If your answer is correct, go on to the next statement; if your answer is not correct, review the section of the text that explains the idea on which the statement is based.

1. In making a comparison we must use ________ numbers.

(two)

2. When you say that one quantity is so much greater than, or so much less than, another quantity, you are comparing the numbers by ______________.

(subtraction)

3. When you say that the population of one city is twice the population of another city, you are comparing numbers by ___________.

(division)

4. The indicated division of one quantity by another is called the _________ of the quantities.

(ratio)

5. Only ________ numbers are compared by a ratio.

(two)

6. The two ways to write the ratio of 5 to 6 are ________ and ________.

(5:6) ($\frac{5}{6}$)

7. The terms of a ratio (do, do not) need to be whole numbers.

(do not)

### Practice Problems

Write each of the following ratios first with a colon and then in the form of a fraction. Compare your answers with the answers given at the back of this text.

1. 6 ft (feet) to 3 ft
2. \$7 to \$4
3. 9 lb (pounds) to 27 lb
4. 20 min (minutes) to 30 min
5. 18 cm (centimeters) to 8 cm
6. 14 cu ft (cubic feet) to 21 cu ft

### Value of a Ratio

8. The value of a ratio is the value of the fraction that you get when you write the ratio as a fraction. This value may be indicated by merely writing the fraction, or it may be expressed as the result of dividing the first term by the second term. When you merely indicate the division, the value of the ratio may be either a proper fraction or an improper fraction. When you actually perform the division, the quotient may be a whole number, a mixed number, a decimal, or a mixed decimal.

The best way to write the value of a ratio depends not only on the numbers in the ratio, but upon how the ratio is to be used. For example, the value of the ratio 30:12 may be written in any of the following ways: as the fraction $\frac{30}{12}$; as any equivalent fraction, such as $\frac{10}{4}$ or $\frac{5}{2}$; as the mixed number $2\frac{1}{2}$; or as the mixed decimal 2.5. The value of the ratio 2:3 would usually be written as the fraction $\frac{2}{3}$, but it could be written as the decimal 0.667. The value of the ratio 6.83:4.17 would usually be written either as the fraction $\frac{6.83}{4.17}$ or as the mixed decimal 1.638

or 1.64. The value of the ratio $\frac{3}{4}:\frac{5}{12}$ is the quotient obtained by dividing the fraction $\frac{3}{4}$ by the fraction $\frac{5}{12}$. This quotient is found by multiplying $\frac{3}{4}$ by the reciprocal of $\frac{5}{12}$. Thus the value of this ratio is $\frac{3}{4} \times \frac{12}{5}$, which is $\frac{9}{5}$ or 1.8.

### Quantities in a Ratio

9. The two numbers or quantities being compared in a ratio must be of a similar nature. Thus, if one quantity is a distance, the other quantity must also be a distance; if one is a weight, the other must be a weight; if one is an amount of time, the other must be an amount of time, and so on. Remember this fact and you will avoid many errors in using ratios to solve problems.

Furthermore, since the value of a ratio is found by dividing the numerator of a fraction by its denominator, both terms must be expressed in the same units. Therefore, when you write a ratio, be certain that both terms are quantities of the same kind and that both are expressed in the same units. For example, suppose you know that a distance is 30 in. and another distance is 1 ft. To write the ratio of the first distance to the second, you should express both distances in inches or both in feet. Thus you may write this ratio either as 30 in.:12 in. or as $\frac{30}{12}$ ft:1 ft.

The value of the ratio 30 in.:12 in. is $30 \div 12 = 2.5$, and the value of the ratio $\frac{30}{12}$ ft:1 ft is $\frac{30}{12} \div 1$, which is equal to $\frac{30}{12}$ or 2.5. Notice that the value of the ratio 30 in.:12 in. is the same as the value of the ratio $\frac{30}{12}$ ft:1 ft. It is a fact that you will always get the correct value of the ratio whether the distances are expressed in inches or in feet. Just remember that both terms must be expressed in the same units.

## Check Your Learning

8. The value of the ratio 4:5 can be written as ________ or ________.
($\frac{4}{5}$) (0.8)

9. The value of the ratio $\frac{1}{2}:\frac{3}{4}$ is ________.
($\frac{2}{3}$)

10. The two quantities compared in a ratio must be expressed in the same ________.
(units)

## Practice Problems

Two quantities are given in each of the following problems. By using a colon, write the ratio of the first quantity to the second quantity in two ways. To answer problem 1, for instance, first express both distances in inches and then express both distances in feet. Express a part of a unit as a fraction reduced to its lowest terms.

1. 3 in. and 1 ft
2. 4 hr (hours) and 30 min
3. 2 doz (dozen) and 16
4. $1\frac{1}{4}$ oz (ounces) and 3 lb
5. 2 sq ft (square feet) and 72 sq in. (square inches)
6. 32¢ and $4

Did you know that the size of our flag is set by law to be 1.9 to 1?

### Equivalent Ratios

10. You will recall that the value of a fraction is not changed when both numerator and denominator are multiplied or divided by the same number. Therefore, you do not change the value of a ratio when you multiply or divide both terms of the ratio by the same number.

For example, consider the ratio 6:24. The value of this ratio, obtained by dividing 6 by 24, is 0.25. If each of the terms of the given ratio, 6 and 24, is multiplied by 3, the resulting ratio is 18:72. The value of this new ratio, obtained by dividing 18 by 72, is 0.25 also. Furthermore, if each term of the ratio 6:24 is divided by 3, the result is the ratio 2:8. When you divide 2 by 8, the quotient is 0.25, just as it was before. So each of the three ratios 6:24, 18:72, and 2:8 has the same value.

Two ratios are *equivalent* when they have the same value. If the terms of one ratio can be made equal to the terms of another ratio by multiplying or dividing them by the same number, the two ratios are equivalent.

Previously we showed that the value of the ratio of one distance to another distance is the same whether the distances were expressed in inches or in feet. In that example you saw that the ratio 30 in.:12 in. is equivalent to the ratio $\frac{30}{12}$ ft:1 ft. Observe that the terms of the second ratio can be obtained by dividing the terms of the first ratio by 12. In other words, 30:12 becomes $\frac{30}{12}:\frac{12}{12}$ or $\frac{30}{12}$:1.

### Ratio with 1 as the Smaller Term

11. In some work it is the usual practice to express all ratios so that the smaller term of each ratio is 1. Either the first term or the second term may be 1. Examples are 1:3 and $1\frac{1}{2}$:1. If neither term of a ratio is 1, you can find the equivalent ratio in which the smaller term is 1 by proceeding as in the following examples. To check your result, you should find the value of each ratio by dividing the first term by the second term. Of course, the two quotients, or the values of the ratios, should be the same.

EXAMPLE 1. A ratio is 12:3. Write an equivalent ratio in which the smaller term is 1.

SOLUTION. The first term of the ratio is greater than the second term, so the first term of the desired ratio must be greater than the second term. The smaller term in the new ratio will, therefore, be the second term.

To obtain 1 as the second term in the new ratio, as required, you must divide the second term by itself. Then, so that the new ratio will be equivalent to the given ratio, you must divide its first term by the same number. Therefore, the first term of the new ratio is found by dividing the first term of the given ratio by 3. The desired ratio is 4:1. Ans.

As a check, $12 \div 3 = 4$ and $4 \div 1 = 4$.

EXAMPLE 2. You have to replace the ratio 2:3 by another ratio in which the smaller term is 1. What is the new ratio?

SOLUTION. The smaller term of the given ratio is the first term. Therefore, the first term of the desired ratio must be 1. To obtain 1 as the first term in the new ratio, you must divide the first term in the given ratio, which is 2, by itself. The second term in the new ratio is the quotient obtained by dividing 3 by 2. Since this quotient is $1\frac{1}{2}$ or 1.5, the desired ratio is 1:$1\frac{1}{2}$ or 1:1.5. Ans.

As a check, $2 \div 3 = 0.667$ and $1 \div 1.5 = 0.667$.

EXAMPLE 3. A ratio is given as 4:$\frac{1}{2}$. What is an equivalent ratio having 1 as the smaller term?

SOLUTION. In this example the smaller term is the second term. To obtain 1 as the second term in the new ratio, you must multiply $\frac{1}{2}$ by 2; so you find the first term in the new ratio by multiplying the first term by 2. Since this product is 8, the desired ratio is 8:1. Ans.

As a check, $4 \div \frac{1}{2} = 8$ and $8 \div 1 = 8$.

### Check Your Learning

11. You do not change the ____________ of the ratio when you multiply or divide both terms of the ratio by the same number.

(value)

12. Two ratios are ____________ when they have the same value.

(equivalent)

13. To obtain 1 as a term of a ratio, you must divide the term by __________.

(itself)

14. When you multiply one term of a ratio by a number, you must multiply the other term by the __________ number.

(same)

## Practice Problems

1. Find the value of each of the following ratios. If the value is not a whole number, express it as a proper or an improper fraction. Be sure to reduce it to its lowest terms.

   *a)* 72:8 *b)* 19:76 *c)* 60:35 *d)* 120:18

2. Express the value of each of the following ratios as a decimal or a mixed decimal. Carry the result to three decimal places:

   *a)* 6.25:0.75 *b)* 48:216 *c)* 435:450

3. For each of the following ratios, write an equivalent ratio in which the smaller term is 1. If the larger term is not a whole number, express it as a mixed decimal.

   *a)* 30:6 *b)* 4:7 *c)* 5:2 *d)* $\frac{1}{4}$:2

## *Proportion*

### What a Proportion Is

12. As explained in Art. 10, two ratios that have the same value are equal or equivalent. When you know that two ratios are equal, you can write an equation by putting an equality sign between the two ratios. For example, the ratio 3:6 is equivalent to the ratio 12:24 because each has the same value, $\frac{1}{2}$. Therefore, you can write the equation $3:6 = 12:24$. This equation, which is a statement of equality between two ratios, is called a *proportion*. Thus $3:6 = 12:24$ is a proportion.

In $3:6 = 12:24$, notice that each ratio is written with a colon. Each ratio can also be written in the form of a fraction. Thus the equation $\frac{3}{6} = \frac{12}{24}$ can be considered a proportion.

Some people write a proportion by using a double colon (::) instead of the equality sign. For example, the proportion involving the ratios 3:6 and 12:24 can be written like this:

$$3:6::12:24$$

In this text, we shall write each ratio in the form of a fraction and use an equals sign between the ratios. We hope that doing this will make it easier for you to see what is done.

### Parts of a Proportion

13. Every proportion contains two ratios that have the same value. The ratio to the left of the equals sign is called the first ratio, and the ratio to the right of the equals sign is called the second ratio. For instance, in $3:6 = 12:24$ the first ratio is 3:6 and the second ratio is 12:24. And in the proportion $\frac{3}{6} = \frac{12}{24}$ the first ratio is $\frac{3}{6}$ and the second ratio is $\frac{12}{24}$.

Since each ratio has two terms, a proportion has four terms. These terms are often called the first, second, third, and fourth terms. The first term is the one at the left, and the fourth term is the one at the right. Thus, in the proportion $3:6 = 12:24$, the first term is 3, the second term is 6, the third term is 12, and the fourth term is 24. The same thing is true if the proportion is written as $\frac{3}{6} = \frac{12}{24}$.

The first and fourth terms of a proportion are sometimes called the *extremes*, and the second and third terms are called the *means*. Thus, in the proportion $3:6 = 12:24$, the numbers 3 and 24 are the extremes and 6 and 12 are the means.

### Check Your Learning

15. An equation which is a statement of equality between two ratios is called a ________________ .

(proportion)

16. In a proportion ratios may be written as ________________ .

(fractions)

17. A proportion has ________ terms.

(four)

18. The first and fourth terms of a proportion are called the ____________.

(extremes)

19. The second and third terms are called the ____________.

(means)

20. A proportion may be written by using a double ____________ instead of the equality sign.

(colon)

**Reading a Proportion**

14. There are two common ways to read a proportion. To illustrate these ways to read a proportion, we shall use the proportion written as $3:6 = 12:24$ or as $\frac{3}{6} = \frac{12}{24}$. One way to read these proportions is to say, "The ratio of three to six equals the ratio of twelve to twenty-four." However, the easiest and most common way is to say, "Three is to six as twelve is to twenty-four."

### Practice Problems

1. Write each of the following statements as a proportion by using fractions and an equality sign:
   *a)* The ratio of 5 to 9 equals the ratio of 15 to 27.
   *b)* The ratio of 3 to 0.25 equals the ratio of 12 to 1.
   *c)* The ratio of 4.5 to 6 equals the ratio of 1.5 to 2.
2. By using colons and an equality sign, write a proportion for each of the following pairs of ratios (the semicolons separate the ratios of a pair):
   *a)* 2 and 3; 6 and 9
   *b)* 17 and 15; 68 and 60
   *c)* $\frac{1}{2}$ and $\frac{3}{8}$; 4 and 3
   *d)* 1 and 3; $\frac{1}{3}$ and 1
3. How would you read each of the proportions in problem 2?

**Requirements of a Proportion**

15. Before we continue our study of ratio and proportion by considering some of the applications, let's review what we have learned so far. First, we learned that a proportion is an equality between two ratios, and that the two ratios must therefore have the same value. Second, we learned that the terms of a ratio should be expressed in the same units; hence, the first and second terms of a proportion, and the third and fourth terms, should be in like units. These important points concerning a proportion can be summarized into the following requirements:

1. The value of the first ratio must be the same as the value of the second ratio.
2. The units used in the first and second terms should be alike, and the units used in the third and fourth terms should also be alike.

For a proportion to satisfy the first requirement, either

*a)* The first term must be smaller than the second term, and the third term must be smaller than the fourth term.

*b)* The first term must be greater than the second term, and the third term must be greater than the fourth term.

The proportion 3:6 = 12:24 meets this requirement. So does 6:3 = 24:12. However, be careful not to reverse the terms of only one of the ratios. If you write 3:6 = 24:12 or 6:3 = 12:24, you are saying that two unequal quantities are equal. These quantities are not equal, because the values of the ratios are not the same.

For a proportion to meet the second requirement, the first and second terms must represent quantities of the same kind. Likewise, the third and fourth terms must represent quantities of the same kind. However, the units used for the third and fourth terms need not be the same as the units for the first and second terms. Assume, for example, that the terms of one ratio are 3 hr and 6 hr and the terms of another ratio are $12 and $24. It would be correct to write these ratios in a proportion as 3 hr:6 hr = $12:$24. But do not write 3 hr:$12 = 6 hr:$24. Observe that this last relation meets the first requirement, because the value of 3:12 is 0.25 and the value of 6:24 is 0.25 also. But it does not meet the second requirement. Notice how important both requirements are! Unless you consider the units as well as the numbers when you write a proportion, you may make a mistake in solving a problem in which the proportion is used.

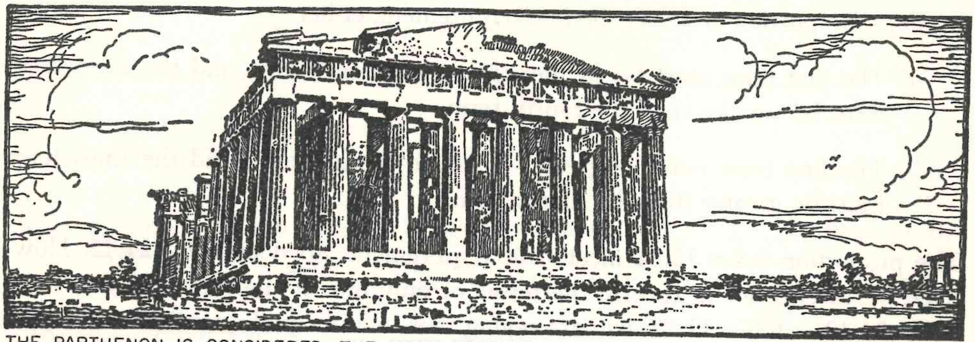
### How Proportions Are Used to Solve Problems

16. Whenever you use a proportion to solve a problem, you will know three of the terms and you will have to find the other term. That is, you will know the two terms of one ratio and one term of a second ratio. From these facts you will have to find the other term of the second ratio. To find this term, you will have to write the proportion correctly. This is what you are going to learn to do now. We shall give some examples showing the procedure of writing a proportion to express the conditions of a problem. Later on, we shall show you how to find the value of this term.

Since you will have to write a proportion before you know one of the terms, you must use a symbol to represent the unknown term. The usual practice is to use $x$ for the unknown term. When you start to work a problem in which you must write a proportion having an unknown term, it is a good idea to start with a statement like this:

Let $x$ = the required distance, in feet

This is just an illustration. You would, of course, substitute other words for "distance" and "feet" depending upon the conditions of the problem.



THE PARTHENON IS CONSIDERED THE MOST PERFECTLY PROPORTIONED BUILDING IN THE WORLD. THE RELATION BETWEEN ITS HEIGHT AND WIDTH IS APPROXIMATELY TWO TO THREE.

## How to Write a Proportion When One of the Ratios Is Given

17. In some problems the two terms of one ratio will be numbers without units. You will be able to tell from the conditions of the problem whether the unknown term is to be greater or less than the other quantity given in the problem. Here are two examples illustrating this kind of problem:

EXAMPLE 1. The ratio of one distance to another is to be 2:3, and the smaller distance is to be 8 in. Write a proportion that includes the longer distance.

SOLUTION. The given information may be stated in the following manner: The given distance, 8 in., is to the unknown, or required, distance as 2 is to 3. Before you try to express this statement by writing a proportion, denote the required distance by the symbol $x$ and write:

Let $x =$ the required distance, in inches

The proportion then becomes:

$$8:x = 2:3 \quad \text{or} \quad \frac{8}{x} = \frac{2}{3} \quad \text{Ans.}$$

Surely, you can see how the proportion was obtained in this example. Now let's check the terms to see if the result meets the two requirements of a proportion stated in Art. 15. The third term, 2, is smaller than the fourth term, 3. To meet the first requirements, the first term, 8, must be smaller than the second term, $x$. Since 8 in. represents the smaller of the two distances, the first requirement is satisfied.

For the proportion to meet the second requirement, the first term and the second term must be expressed in units having the same name. In this example 8 is a number of inches and $x$ represents a number of inches also. The third and fourth terms are merely numbers without units; hence, the second requirement is satisfied.

EXAMPLE 2. In a mixture of two materials, the ratio of the volume of one material to the volume of the other material is to be 7:4. You are to use 91 gal (gallons) of the first material. Write a proportion that includes the volume of the other material.

SOLUTION. You know that 91 gal is to the smaller volume as 7 is to 4. Therefore, you should write:

Let $x =$ required volume, in gallons

Next, you write the proportion as follows:

$$91:x = 7:4 \qquad \text{or} \qquad \frac{91}{x} = \frac{7}{4} \qquad \text{Ans.}$$

Let's check this proportion to see if it meets the requirements of a proportion. Since the third term is greater than the fourth term, the first term must be greater than the second term. We know that 91 gal will be greater than $x$, so the first requirement is met. The first term is in gallons, and the second term, $x$, is also in gallons. Again in this example, the third term and the fourth term are numbers without units. Therefore, the second requirement is met.

### Check Your Learning

21. In a proportion the ____________ of the two ratios must be equal.

(values)

22. In a proportion the ____________ used in the first two terms must be alike and the ____________ used in the last two terms must be alike.

(units) (units)

23. The units used for the third and fourth terms (need, need not) be the same as the units used for the first and second terms.

(need not)

24. If in a proportion the first term is larger than the second term, then the third term must be ____________ than the fourth term.

(larger)

25. If the second term of a proportion is larger than the first term, then the ____________ term must be larger than the ____________ term.

(fourth) (third)

### How to Write a Proportion When Both Ratios Must Be Formed

18. Sometimes when a proportion is used to solve a problem, all four terms will be quantities expressed in units. Two terms will refer to one property of a particular thing, and the other terms will refer to a different property of the same thing. When we speak of a property of a thing, we may mean its length, its area, its volume, its weight, its cost, the time needed to do work on it, or some other quantity relating to it. For example, we can write a proportion in which two terms are the weights of two materials and the other two terms are the costs of the materials. Or we can write a proportion in which two terms refer to the number of men in two groups that might be assigned to do a job and the other two terms refer to the number of hours needed to do the work.

Before you can write a proportion in which the terms refer to two properties of a certain thing, you must answer these questions: 1) Does the quantity measuring one property increase as the quantity measuring the other property increases? 2) Does the quantity measuring one property decrease as the quantity measuring the other property increases? In the following examples we shall show you how the answers to these questions are used in writing a proportion.

EXAMPLE 1. Suppose the cost of 7 lb of putty is 56 cents and you want to know the cost of 36 lb. Write the proportion that will include the cost of 36 lb.

SOLUTION. The terms of one ratio are the two weights, and the terms of the other ratio are the two costs. You know that 36 lb of putty will cost more than 7 lb because the cost increases as the weight increases. For this reason, you can say that 7 lb is to 36 lb as 56 cents is to the greater cost. Before you attempt to write the proportion, don't forget to write:

Let $x =$ cost of 36 lb, in cents

The proportion then becomes

$$7:36 = 56:x \qquad \text{or} \qquad \frac{7}{36} = \frac{56}{x} \qquad \text{Ans.}$$

You should examine this result to see that it meets both requirements of a proportion.

EXAMPLE 2. Eight men took 32 hr to complete a job. You want to know how many hours will be needed for a similar job if only five men are available. Assume that each man will do the same amount of work in an hour, and write a proportion that will include the number of hours needed by the five men.

SOLUTION. The terms of one ratio should be the numbers of men. The terms of the other ratio should be the numbers of hours. Common sense tells you that the five men will need more time than the eight men. So you can say that 8 is to 5 as the greater number of hours is to 32. Therefore, you should write:

Let $x =$ number of hours needed by five men

The proportion is

$$8:5 = x:32 \qquad \text{or} \qquad \frac{8}{5} = \frac{x}{32} \qquad \text{Ans.}$$

Don't forget that it is very important to put the terms of the proportion in the correct positions.

### How to Arrange the Terms in a Proportion

19. In each of the examples in Arts. 17 and 18, the terms of the proportion were written in the order that agreed with the statement of the problem. However, there are different ways in which the terms of any proportion can be arranged. For example, any of the following forms would be correct for the proportion given in example 1 of Art. 17:

$$8:x = 2:3 \qquad \text{or} \qquad \frac{8}{x} = \frac{2}{3}$$

$$2:3 = 8:x \qquad \text{or} \qquad \frac{2}{3} = \frac{8}{x}$$

$$3:2 = x:8 \qquad \text{or} \qquad \frac{3}{2} = \frac{x}{8}$$

$$x:8 = 3:2 \qquad \text{or} \qquad \frac{x}{8} = \frac{3}{2}$$

In example 2 of Art. 17 these are the possible arrangements:

$$91:x = 7:4 \qquad \text{or} \qquad \frac{91}{x} = \frac{7}{4}$$

$$7:4 = 91:x \qquad \text{or} \qquad \frac{7}{4} = \frac{91}{x}$$

$$4:7 = x:91 \qquad \text{or} \qquad \frac{4}{7} = \frac{x}{91}$$

$$x:91 = 4:7 \qquad \text{or} \qquad \frac{x}{91} = \frac{4}{7}$$

In a similar manner you can write the proportions of examples 1 and 2 in four different ways. You may wonder which of these four ways of writing a proportion is the best. Actually, there is no "best" way, because each way is just as good as any other. In other words, any one of these four ways can be used. To help you in deciding which form to use, we shall always write the unknown term as the fourth term. There are several good reasons for writing a proportion in this way. Probably the best reason for it is to eliminate all the other ways. And furthermore, we know from experience that you will find the reasoning easier to follow when the unknown is the fourth term.

EXAMPLE 1. Write the proportion in example 1 of Art. 17 so that $x$ will be the fourth term. In this example the ratio of one distance to another is 2:3 and the smaller distance is 8 in.

SOLUTION. Remember that the third term and the fourth term of a proportion must be quantities expressed in the same units. In this example the unknown fourth term is a distance. Since the third term must also be a distance, it must be 8 in. Furthermore, according to the conditions in this example, 8 must be smaller than $x$.

To meet the first requirement of a proportion, the first term must be smaller than the second. So the first ratio must be 2:3. Hence, the desired proportion is

$$2:3 = 8:x \qquad \text{or} \qquad \frac{2}{3} = \frac{8}{x} \qquad \text{Ans.}$$

EXAMPLE 2. Write the proportion in example 2 of Art. 17 so that $x$ is the fourth term. In this example the ratio of one volume to another is 7:4 and the greater volume is 91 gal.

SOLUTION. The unknown term, which is to be the fourth term, is a volume. This means that the third term must be a volume also, and the second ratio is $91:x$. According to the conditions of this example, 91 must be greater than $x$; therefore, the first term of the proportion must be greater than the second term. Hence the first ratio is 7:4. The desired proportion is

$$7{:}4 = 91{:}x \qquad \text{or} \qquad \frac{7}{4} = \frac{91}{x} \qquad \text{Ans.}$$

**EXAMPLE 3.** Write the proportion in example 1 of Art. 18 so that $x$ will be the fourth term. In this example the cost of 7 lb of putty is given as 56 cents and you want to know the cost of 36 lb.

**SOLUTION.** Since the fourth term, $x$, of the proportion represents a cost, the third term must be the given cost, 56 cents. You know that the cost of putty increases as its weight increases. Here the third term is smaller than the fourth term, $x$, so the first term must be smaller than the second term. The desired proportion is

$$7{:}36 = 56{:}x \qquad \text{or} \qquad \frac{7}{36} = \frac{56}{x} \qquad \text{Ans.}$$

**EXAMPLE 4.** Write the proportion in example 2 of Art. 18 with $x$ as the fourth term. In this example eight men need 32 hr to complete a certain job, and you want to know how many hours five men will need.

**SOLUTION.** The fourth term, $x$, of the proportion represents a number of hours. The third term must therefore be 32 hr. You know that the number of hours required to do the work will decrease as the number of men increases and will increase as the number of men decreases. The third term in this example must, therefore, be smaller than $x$. To make the first term smaller than the second term, the proportion must be

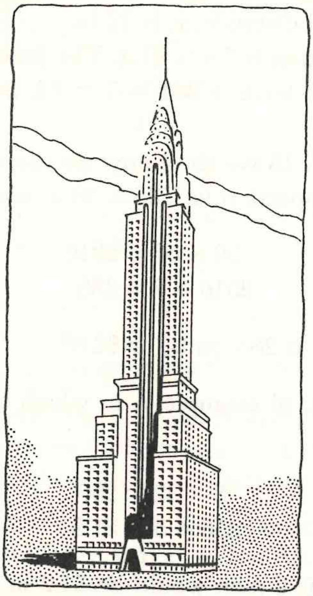
$$5{:}8 = 32{:}x \qquad \text{or} \qquad \frac{5}{8} = \frac{32}{x} \qquad \text{Ans.}$$

### Practice Problems

In each of the following problems write a proportion in which the quantity to be found is the fourth term. Denote this term by $x$.

1. The ratio of one angle to another is to be 5:4, and the larger angle is to be 75° (degrees). Find the smaller angle.
2. The cost of 25 cu yd (cubic yards) of concrete is $650. Find the cost of 48 cu yd.
3. You can travel 24 miles in 39 min. You want to know the number of minutes you will need to go 52 miles if you travel at the same rate.
4. A company of 200 men has enough food for 8 days. You want to know how many days the food will last if the number of men is increased to 240.

240:200 = 8:x



The architect uses ratio and proportion to design buildings.

## *Applications of Proportion*

### Important Relation Between Terms of a Proportion

20. If you examine any proportion, you will observe that the product of the first and fourth terms is equal to the product of the second and third terms. This relation is often stated in the following rule: *In any proportion the product of the extremes is equal to the product of the means.*

In the proportion $3:6 = 12:24$, for example, the product of the extremes is $3 \times 24 = 72$. The product of the means, $6 \times 12$, is also 72. You should check the truth of this important rule. You can test it for yourself by comparing these two products for any proportion in which all four terms are known. As we said before, this is a very important relation. It is also a very useful relation, because by means of it we can find the unknown term in a proportion.

### Finding the Unknown Term in a Proportion

21. When three terms of a proportion are known and the other term is unknown, there is a simple way to find the missing term. Here it is:

Find the product of the second and third terms and divide it by the first term. Don't forget that the fourth term is the unknown term, $x$. We shall now show you how to use this method to find the fourth term of the proportion in each example in Art. 17.

In example 1 the proportion is $2:3 = 8:x$. The product of the second term, 3, and the third term, 8, is 24, and the quotient obtained by dividing 24 by the first term, 2, is 12. So the desired distance, $x$, is 12 in.

In example 2 the proportion is $7:4 = 91:x$. The product of the second and third terms is 364, and the fourth term is $364 \div 7 = 52$. So the desired volume, $x$, of material is 52 gal.

For the examples in Art. 18 we shall now do the same thing. The solution of example 1, in which the proportion is $7:36 = 56:x$, is as follows:

$$36 \times 56 = 2016$$
$$2016 \div 7 = 288$$

The cost $x$ of 36 lb of putty is 288 cents, or \$2.88.

The steps in the solution of example 2, in which the proportion is $5:8 = 32:x$, are as follows:

$$8 \times 32 = 256$$
$$256 \div 5 = 51.2$$

The time $x$ needed by the five men would be 51.2 hr, or 51 hr 12 min.

Here are the complete solutions of a few more examples:

EXAMPLE 1. A pump discharges 30,000 gal of water in 12 hr. How many hours will be needed to pump 120,000 gal?

SOLUTION. The fourth term, $x$, of the proportion will be a number of hours; so the third term must be the given number of hours, which is 12. When you analyze the problem, reason tells you that more time will be needed to pump 120,000 gal than to pump 30,000 gal. Therefore, the fourth term must be greater than the third term, and the second term must be greater than the first term. The required proportion is

$$30{,}000:120{,}000 = 12:x \qquad \text{or} \qquad \frac{30{,}000}{120{,}000} = \frac{12}{x}$$

To find $x$, multiply the second term of the proportion by the third term and then divide this product by the first term. Thus,

$$120{,}000 \times 12 = 1{,}440{,}000$$
$$1{,}440{,}000 \div 30{,}000 = 48$$

So the time, $x$, is 48 hr. Ans.

EXAMPLE 2. A metal bar 3 ft 6 in. long weighs 11.90 lb. What will be the weight of a piece of the bar 9 in. long?

SOLUTION. The fourth term, $x$, of the proportion will be a number of pounds. The third term must therefore be 11.90. The weight of the piece of the bar will be less than the weight of the whole bar, so $x$ will be less than 11.90, and the second term must be less than the first term. Both lengths must be expressed in inches or both must be expressed in feet. We shall change 3 ft 6 in. to 42 in., and the proportion is

$$42:9 = 11.90:x \qquad \text{or} \qquad \frac{42}{9} = \frac{11.90}{x}$$

The required weight, $x$, is found as follows:

$$9 \times 11.90 = 107.10$$
$$107.10 \div 42 = 2.55$$

The weight, $x$, of the piece of bar will be 2.55 lb. Ans.

For your own practice you should solve this example by expressing both lengths in feet. Your proportion should look like this:

$$3.5:\tfrac{3}{4} = 11.90:x \qquad \text{or} \qquad \frac{3.5}{0.75} = \frac{11.90}{x}$$

Your answer should, of course, be the same one that we obtained previously.

**EXAMPLE 3.** A machine can dig 5 cu yd of earth while another machine digs 3.5 cu yd. If the first machine needs 20 hr to remove a certain volume of earth, how much time would the second machine need to remove the same volume?

**SOLUTION.** Here the fourth term, $x$, will be a number of hours, so the third term must be 20. Obviously, the second machine will require more time than the first machine. The fourth term, $x$, must therefore be greater than the third term. This means that the second term of the proportion must be greater than the first term. The proportion is

$$3.5:5 = 20:x \qquad \text{or} \qquad \frac{3.5}{5} = \frac{20}{x}$$

The value of $x$ is found as follows:

$$5 \times 20 = 100$$
$$100 \div 3.5 = 28.57$$

In an example of this kind the time $x$ would probably be rounded off to $28\frac{1}{2}$ hr instead of being called 28.57 hr, or 28 hr 34 min. Ans.

## Check Your Learning

26. There are __________ possible ways to write a proportion.

(four)

27. In this text the unknown term will be the __________ term.

(fourth)

28. In any proportion, the product of the extremes is equal to the product of the __________.

(means)

## Practice Problems

Find the value of $x$ in each of the practice problems in Art. 19.

## Review Problems

1. If a pump that discharges 6 gal of water per minute can fill a tank in 20 hr, how long will a pump that discharges 15 gpm (gallons per minute) take to fill it?
2. If 75 lb of metal costs $6.75, how much will 125 lb cost?
3. If A does as much work in 4 days as B does in 7 days, how long will it take A to do what B can do in 63 days?
4. If a car goes 12 miles in 35 min, how long will it take to go 30 miles at the same rate?
5. A man working 8 hr each day requires 12 days to complete a job. How many days will he require to complete the job if he works 10 hr each day?
6. The relative speeds of two automobiles are as 3:2. How many hours will be required by the slower automobile to travel over the distance covered by the faster automobile in 6 hr?
7. A company of 200 men has provisions sufficient to last 8 days. If a reinforcement of 40 men arrives, how long will the provisions last?
8. A pump making 33 strokes per minutes discharges 660 gal of water per minute. If the speed is increased to 36 strokes per minute, what will be the discharge, in gallons per minute?
9. If 4 yd of dress fabric costs $6.75, how much will $6\frac{1}{2}$ yd cost?
10. If a block of stone 6 ft long, 3 ft wide, and 1.5 ft thick weighs 4500 lb, what is the weight ,to the nearest pound, of a block of stone 3 ft long, 1 ft wide, and 2 ft thick?

### Proportional Quantities

22. In some of the preceding examples, the quantity measuring one property of a thing increased as the quantity measuring another property increased. In other problems the quantity measuring one property decreased as the quantity measuring another property increased. You were able to determine by reasoning which condition occurred in these cases. However, there will be times in practical work when you will be told whether the quantity measuring one property increases or decreases as the quantity measuring another property increases. The expressions *directly proportional* and *inversely proportional* are often used for this purpose.

When one property increases as another property increases, we say that the first property is *directly proportional* to the second property. For instance, in example 1 of Art. 18 the cost of putty increased as the weight of putty increased. So the cost of putty is *directly proportional* to its weight. In other words, the cost becomes greater as the weight becomes greater or it becomes smaller as the weight decreases.

To sum this up, if you are told that the cost of putty is directly proportional to its weight, you should understand that the cost of putty *increases as the weight* *of putty increases* and that the cost of putty *decreases as the weight of putty decreases.*

When one property decreases and another property increases, the first property is inversely proportional to the second property. In example 2 of Art. 18 the number of hours needed to do the work increased as the number of men decreased. It follows also that the number of hours would decrease as the number of men increased. So it may be said that the time needed to do a certain amount of work is *inversely proportional* to the number of men working. In other words, the number of hours needed becomes smaller as the number of men becomes greater or the time becomes greater as the number of men decreases.

Again in summary, if you are told that the time required to do a certain amount of work is inversely proportional to the number of men working, you should understand that the *amount of time decreases as the number of men increases* and that the *amount of time increases as the number of men decreases.*

The word "proportional" may be used alone instead of the expression "directly proportional." That is, you may omit the word "directly" when you mean "directly proportional." For example, you may say that the cost of putty is proportional to its weight. However, never omit the word "inversely" when you mean "inversely proportional."

EXAMPLE 1. You are told that the areas of two circles are proportional to the squares of their radii. If the area of a circle having a radius of 2 in. is 12.56 sq in., what is the area of a circle having a radius of 3 in.?

SOLUTION. Since the word "proportional" is used alone, it means that the quantities are directly proportional; so the area of the circle with the larger radius is greater than the area of the circle with the smaller radius. The fourth term, $x$, of the desired proportion will be the area of the larger circle. The third term must, therefore, be the area of the smaller circle. Since $x$ is larger than the third term, the second term of the proportion must be larger than the first term. The proportion is written in this way:

$$2^2:3^2 = 12.56:x \qquad \text{or} \qquad \frac{2^2}{3^2} = \frac{12.56}{x}$$

These proportions become

$$4:9 = 12.56:x \qquad \text{or} \qquad \frac{4}{9} = \frac{12.56}{x}$$

The calculations for finding $x$ are as follows:

$$9 \times 12.56 = 113.04$$
$$113.04 \div 4 = 28.26$$

The required area, $x$, is 28.26 sq in. Ans.

EXAMPLE 2. The number of hours of traveling time required to haul a certain volume of earth from one place to another in trucks is inversely proportional to the size of the trucks used. If 5-ton trucks are used, the hauling will take 130 hr. How many hours (to the nearest hour) will the hauling job take if 3-ton trucks are used?

**Solution.** The number of hours in inversely proportional to the size of the trucks hence, the required number of hours will be greater when the smaller trucks are used The fourth term, $x$, will be the number of hours needed by the 3-ton trucks. The third term must be the number of hours needed by the 5-ton trucks. Since $x$ must be larger than the third term, the second term must be larger than the first term. The proportion and the calculations are as follows:

$$3:5 = 130:x \quad \text{or} \quad \frac{3}{5} = \frac{130}{x}$$

$$5 \times 130 = 650$$

$$650 \div 3 = 216\tfrac{2}{3}$$

The required time, to the nearest hour, is 217 hr. Ans.

## Check Your Learning

29. When one property increases as the other increases, we say the properties are __________ proportional.

(directly)

30. If one property increases as the other decreases, we say these properties are __________ proportional.

(inversely)

31. Sometimes the word "directly" (may, may not) be omitted.

(may)

32. The word "inversely" (may, may not) be omitted.

(may not)

33. When one quantity decreases as another decreases, the quantities are __________ proportional.

(directly)

## Practice Problems

1. The weight of cast-iron pipe of a certain diameter is directly proportional to its length. If a piece 12 ft long weighs 485 lb, what will be the weight, to the nearest pound, of a piece 4 ft 10 in. long?
2. The electrical resistance of copper wire is inversely proportional to the cross-sectional area of the wire. The resistance of a certain length of wire having a cross-sectional area of 0.00204 sq in. is 12.21 ohms (the ohm is a unit of electrical resistance). What will be the resistance, to the nearest hundredth of an ohm, of the same length of wire having a cross-sectional area of 0.00322 sq in.?
3. The wind pressure on a flat surface is proportional to the square of the speed of the wind. The pressure on a certain surface is 320 lb when the wind is blowing at 25 miles an hour. What will be the pressure on the surface, to the nearest pound, when the wind blows at 40 miles an hour?
4. A fact given in geometry is that the areas of two similar plane figures are propor-

tional to the squares of a corresponding side of each. Assume that $A$ and $B$ represent the areas and $a$ and $b$ represent the corresponding sides, respectively. Write a proportion based on the fact given above.

## *The Unit Method*

### Solving Problems by the Unit Method

23. All of the problems involving ratios that you have learned to solve by proportions can be solved by another method. This new way to solve such problems is called the *unit method.* We are going to explain this method and show how to use it because some persons find it easier to understand and because it makes it possible to work problems in which more than two ratios are involved. The procedure is outlined in the following example:

EXAMPLE 1. Four bags of rock-wool pellets cost \$1.36. How much will 55 bags cost?

SOLUTION. Since the cost of 4 bags = \$1.36, the cost of 1 bag = \$1.36 ÷ 4 = \$0.34. Therefore the cost of 55 bags = 55 × \$0.34 = \$18.70. Ans.

Notice in the preceding example that we used the cost of 4 bags, which was given in the problem, to find the cost of 1 bag. That is, we found the cost of *a unit.* We then used the unit cost to find the cost of the desired number of units.

In this example, we actually calculated the price of a unit. But this is not necessary and it would not be done in practical work. Instead, we would merely indicate the operations of multiplication and division and perform them in the last step. Our work could be indicated as

$$\text{Cost of 55 bags} = 55 \times \frac{\$1.36}{4} = \$18.70$$

It's a good idea to follow this procedure in all of your work in mathematics, because it often simplifies the calculations involved in a solution. In other words, indicate all calculations and do them at the very last. Cultivate the habit of doing this and you will simplify your work considerably and lessen the chance of making errors. In fact, when you have developed this ability to set up a problem and then perform all of the calculations at once, you will have attained a certain degree of mathematical maturity. Strive diligently to learn this method, because it is very valuable.

Here are more examples illustrating the unit method.

EXAMPLE 2. If a pump that discharges 6 gpm (gallons per minute) can fill a water tank in 18 hr, how long will a pump that discharges 14 gpm take to fill the tank?

SOLUTION. The 6-gpm pump requires 18 hr to fill the tank. Therefore, 1 gpm requires $6 \times 18$ hr for filling the tank. Then, 14 gpm requires $\frac{6 \times 18}{14}$ hr for filling th tank, and $\frac{6 \times 18}{14} = 7\frac{5}{7}$ hr. Ans.

In this example the time required for filling the tank at the rate of 1 gpm is not calculated. Instead, it is simply indicated by the product $6 \times 18$ hr. This time is then divided by 14 to find the time required for a discharge of 14 gpm.

EXAMPLE 3. If 4 men earn $465.80 in 7 days, how much can 14 men earn in 12 days?

SOLUTION. 4 men in 7 days earn $465.80. Therefore, 1 man in 7 days earns $\frac{\$465.80}{4}$ and 1 man earns $\frac{\$465.80}{4 \times 7}$ in 1 day. Hence, 1 man earns $\frac{\$465.80 \times 12}{4 \times 7}$ in 12 days and 14 men in 12 days earn

$$\frac{14 \times \$465.80 \times 12}{4 \times 7} = \frac{\overset{2}{\cancel{14}} \times \$465.80 \times \overset{3}{\cancel{12}}}{\cancel{4} \times \cancel{7}} = \$465.80 \times 2 \times 3 = \$2794.80. \quad \text{Ans.}$$

EXAMPLE 4. If 3 men can dig a trench in 20 days, how long will 12 men take to dig the trench?

SOLUTION. Because 3 men dig a trench in 20 days, 1 man can dig it in $3 \times 20$ days, and 12 men can dig it in $\frac{3 \times 20}{12}$ days $= 5$ days. Ans.

## Solving More Difficult Problems by Unit Method

24. Previously we said that the unit method was especially useful when more than two ratios are involved in a problem. Such a problem requires a knowledge of *compound proportions*. However, compound proportions is a topic just a bit beyond the basic objectives of this course. But in spite of the fact that we are not going to discuss compound proportions, we do want to show you how the unit method can be used to solve problems of this type. Study the following examples carefully, because a mastery of this method may help you solve problems that you otherwise would not be able to solve.

EXAMPLE 1. If a block of granite 8 ft long, 5 ft wide, and 3 ft thick weighs 7200 lb, what is the weight of a block of granite 12 ft long, 8 ft wide, and 5 ft thick?

FIRST SOLUTION. A block 8 ft long, 5 ft wide, and 3 ft thick weighs 7200 lb. Therefore, a block 1 ft long, 5 ft wide, 3 ft thick weighs $\frac{7200}{8}$ lb and a block 1 ft long, 1 ft wide, 3 ft thick weighs $\frac{7200}{8 \times 5}$ lb. Furthermore, a block 1 ft long, 1 ft wide, 1 ft thick weighs $\frac{7200}{8 \times 5 \times 3}$ lb.

Hence, a block 12 ft long, 8 ft wide, and 5 ft thick weighs

$$\frac{7200 \times 12 \times 8 \times 5}{8 \times 5 \times 3} = \frac{7200 \times \overset{4}{\cancel{12}} \times \cancel{8} \times \cancel{5}}{\cancel{8} \times \cancel{5} \times \cancel{3}} = 28{,}800 \text{ lb} \quad \text{Ans.}$$

SECOND SOLUTION. Here you compare the volumes of the blocks, in cubic feet.

$$\text{Volume of first block} = 8 \text{ ft} \times 5 \text{ ft} \times 3 \text{ ft} = 120 \text{ cu ft}$$
$$\text{Volume of second block} = 12 \text{ ft} \times 8 \text{ ft} \times 5 \text{ ft} = 480 \text{ cu ft}$$

If 120 cu ft weigh 7200 lb, then 1 cu ft weighs $\frac{7200}{120}$ lb. Therefore, 480 cu ft weigh

$$\frac{480 \times 7200}{120} = 28{,}800 \text{ lb} \quad \text{Ans.}$$

EXAMPLE 2. If 7 cows require 35 bu of feed in 20 days, how many days will 96 bu feed 18 cows?

SOLUTION. 35 bu feeds 7 cows for 20 days. Therefore, 1 bu feeds 7 cows for $\frac{20}{35}$ days, and 1 bu feeds 1 cow for $\frac{7 \times 20}{35}$ days. Therefore, 96 bu feeds 1 cow for $\frac{96 \times 7 \times 20}{35}$ days, and 96 bu feeds 18 cows for

$$\frac{96 \times 7 \times 20}{35 \times 18} = \frac{64}{3} = 21\tfrac{1}{3} \text{ days} \quad \text{Ans.}$$

EXAMPLE 3. If 12 men can assemble 96 trailers in 6 days, how many men will be needed to assemble 64 trailers in 8 days?

SOLUTION. In 6 days 96 trailers can be assembled by 12 men, and in 1 day 96 trailers can be assembled by 6 × 12 men. Furthermore, in 1 day 1 trailer can be assembled by $\frac{6 \times 12}{96}$ men. Therefore, in 8 days, 1 trailer can be assembled by $\frac{6 \times 12}{96 \times 8}$ men. (Note that fewer men are needed to do a certain amount of work in 8 days than would be needed to do it in one day.) Finally, in 8 days, 64 trailers can be completed by

$$\frac{\overset{8}{\cancel{64}} \times 6 \times \overset{1}{\cancel{12}}}{\underset{8}{\cancel{96}} \times \underset{1}{\cancel{8}}} = 6 \text{ men} \quad \text{Ans.}$$

## Check Your Learning

34. In the unit method of solving problems calculations are worked (as you go along, at the end).
(at the end)

35. Cancellation (is, is not) a big help in solving problems by using the unit method.
(is)

36. Doing all calculations at one time in any type of problem will __________ your work and will __________ the chance of making errors.
(simplify) (lessen)

Here are some problems for you to do. See how well you have mastered the unit method.

## Practice Problems

1. If a pump discharges 90,000 gal of water in 20 hr, in how many hours will it discharge 144,000 gal?
2. If 25 men can do a certain job in 30 days, how many days will be required by 35 men to do a job that is one and a half times as big?
3. If $189 is paid for 10 tons of coal, how much must be paid for 27 tons?
4. A pump making 30 strokes per minute discharges 450 gal of water per minute. If the speed is reduced to 28 strokes per minute, what will be the discharge, in gallons per minute?
5. A wheel makes 248 revolutions in 5 min. How many revolutions will it make in 1 hr 20 min?
6. A locomotive runs 2.8 miles in 4 min. How far will it go in 1 hr?
7. If 7 bbl (barrels) of oil cost $164.30, what will 42 bbl cost?
8. If 42 yd of goods are made from 9 lb of yarn, how many yards can be made from 165 lb of a similar yarn?
9. Twelve men can do 25 units of work in 15 days. How many days will be required by 36 men to do 52 units?

## *Per Cent*

### Introduction — Comparing by Hundreds

25. Up to this point we have concentrated all our attention on ratios. We have learned what ratios are and how they are used. Remember that a ratio is the result obtained when two numbers are compared by division. Now we are going to learn about another way to compare numbers by division, and we are going to learn another name for the result of this comparison.

This new topic is based on the fact that 100 is a number that is very easy to work with. It is easy to multiply by 100, and it is easy to divide by 100. Because of this you'll see why 100 is a good number to use as a means of comparing other numbers. Surely you have heard expressions such as these:

Ninety times out of one hundred times he will do it.
Jim got eighty-two right out of a hundred.
Forty-three of every hundred cars built are four-door sedans.

Instead of having to refer specifically to 100 when using it to compare numbers, we use a simpler term. This term is the word *percent* or the words *per cent.* Thus it is not necessary to say things such as "out of one hundred," "by the hundred," or "hundredths," because we can use the term "per cent" instead of these expressions. For example, instead of saying that 20 people out of 100 do a certain thing, it is more usual to say that 20 per cent of the people do it. Also, it is more usual to say that 5 per cent of the pieces produced by a machine were defective than it is to say that 5 pieces out of each 100 were defective. However, when the term "per cent" is used, it is not necessary to refer to exactly 100 people, or 100 pieces, or 100 things of any kind. Any number of people or things may be considered in connection with per cent, as you will see in examples and problems that follow.

### Review of Per Cent

26. In Art. 41 of Part 4 of *Practical Arithmetic* you were introduced to per cent in connection with our discussion of decimals. Be sure to review this explanation of per cent, because you will be expected to know how to change a fraction or a decimal to a per cent. You will also have to know how to write a per cent as a decimal or as a fraction. Probably you will recall that you learned how to find a number that is a certain per cent of another number. This is just one of the several types of problems involving per cent. Actually, there are three types, or kinds, of problems that we shall study. However, before we discuss these problems, we should develop some new ideas.

### Terms Used in Per Cent Problems

27. In every per cent problem there are three numbers. These numbers are

*the per cent, the base,* and *the percentage.* Since it is important to be able to identify these numbers, let's take a very close look at them.

*a)* The per cent. This number is easy to recognize because it is always stated as a per cent either by using the words "per cent" or the symbol %. Very often per cent is called the rate, so you should not be surprised to see either "rate" or "per cent" in a problem. Don't forget that they mean the same thing!

*b)* The base. This is the number to which the per cent or rate is applied. In other words, when you *multiply* a number by a per cent, that number is the base.

*c)* The percentage. This number is the product of the per cent and the base. It is unfortunate that the word *percentage* is used very loosely. In spite of this, you should cultivate the habit of using *percentage* only for the result obtained when you multiply a number (the base) by a per cent. Remember that percentage is an amount – so many pounds, dollars, feet, and so forth. Don't forget that per cent and percentage do not refer to the same thing!

## The Three Kinds of Per Cent Problems

28. We said before that there are three types of problems that involve per cent. We shall now study them one at a time.

*a)* FINDING THE PERCENTAGE WHEN BASE AND PER CENT ARE KNOWN. This is the kind of per cent problem that was discussed previously in Part 4 of *Practical Arithmetic,* but it is important enough to review again carefully. To find the percentage of a number, or of a quantity, you multiply the number by the per cent expressed as a decimal.

EXAMPLE 1. What percentage is obtained by taking 36% of 125?

SOLUTION. First change 36% to a decimal, thus you get 0.36. Then, multiply 125 by 0.36. You will get $125 \times 0.36 = 45$. Therefore, 36% of 125 gives a percentage of 45. Ans.

EXAMPLE 2. From a stock of 300 machines, 76% are sold. What was the percentage sold?

SOLUTION. The number sold was 76% of 300. Expressed as a decimal, 76% = 0.76. Then, $300 \times 0.76 = 228$. The percentage sold was 228 machines. Ans.

EXAMPLE 3. A brass casting weighs 348 lb. The brass is made up of 65% copper and 35% zinc. *a)* What is the percentage of copper in the casting? *b)* What is the percentage of zinc in the casting? Express percentages in pounds to the nearest tenth.

SOLUTION. *a)* The weight of copper is 65% of the whole, or 65% of 348. Expressed as a decimal, 65% = 0.65. The percentage of copper is, then,

$$348 \times 0.65 = 226.2 \text{ lb} \quad \text{Ans.}$$

*b)* The percentage of zinc is

$$348 \times 0.35 = 121.8 \text{ lb} \quad \text{Ans.}$$

Another way to find the percentage of zinc is to subtract the percentage of copper from the weight of the casting, or 348 − 226.2 = 121.8 lb, which is the same answer as before.

*b*) FINDING THE PER CENT WHEN BASE AND PERCENTAGE ARE KNOWN. When several numbers are combined into one number, you are frequently required to find what per cent each number is of the whole number. Actually, these numbers represent percentages and you must determine what per cent of the whole number each of them represents. To find this per cent, divide the percentage by the whole and convert this decimal to a per cent. For example, if 105 lb of copper and 15 lb of tin are melted together to form 120 lb of bronze, you may want to know what per cent of the bronze is copper and what per cent is tin. In this case, the amounts of copper and of tin are percentages; the base is the weight of bronze. The per cents of copper and tin would be found by dividing 105 by 120 and 15 by 120. This is how we would proceed:

$$105 \div 120 = 0.875 = 87.5\%$$
$$15 \div 120 = 0.125 = 12.5\%$$

Some additional examples will now be given.

EXAMPLE 1. What per cent is 16 of 64?

SOLUTION. In this example, 64 is the whole and 16 is the percentage. Then, applying the rule, 16 ÷ 64 = 0.25. Finally, 0.25 = 25%. Therefore, 16 is 25% of 64. Ans.

EXAMPLE 2. In a bronze casting weighing 98 lb there is 83.3 lb of copper. What per cent of the casting is copper?

SOLUTION. Applying the rule, 83.3 ÷ 98 = 0.85, and 0.85 = 85%. Therefore, 85% of the casting is copper. Ans.

EXAMPLE 3. From a stock of 300 motors 228 were sold. What per cent of the stock was sold?

SOLUTION. Following the rule, 228 ÷ 300 = 0.76, and 0.76 = 76%. That is, 76% of the stock was sold. Ans.

EXAMPLE 4. In a community drive the amount pledged at the end of the first week was $318,436.50. If $860,000 is the amount to be raised, what per cent of this was pledged? Express the result to the nearest hundredth of a per cent.

SOLUTION. The problem is to find what per cent $318,436.50 is of $860,000, so divide 318,436.50 by 860,000 to get a decimal, and then convert this decimal to per cent. To compute the per cent to the nearest hundredth, you must round off the decimal obtained by division to four decimal places. This means that you should carry out the division to five places. To simplify the division, move the decimal point four places to the left in both the divisor and the dividend. In doing so, you drop the four ciphers in 860,000 and change the dividend to 31.84365. The division is

$$\begin{array}{rl} & .37027 \\ 86 & \overline{)\,31.84365} \\ & \underline{25\,8}\phantom{4365} \\ & 6\,04\phantom{365} \\ & \underline{6\,02}\phantom{365} \\ & 236\phantom{5} \\ & \underline{172}\phantom{5} \\ & 645 \\ & \underline{602} \end{array}$$

carried out, as shown at the right, and the quotient is rounded off to 0.3703. The amount pledged is 37.03% of the amount to be raised. Ans.

EXAMPLE 5. In 1950 the number of people living in a certain city was 41,514, and in 1960 the number was 68,296. What was the per cent of increase in the population during the 10 years? Express the result to the nearest tenth.

SOLUTION. The increase in population is found by subtracting 41,514 from 68,296. It is 26,782. To find the per cent of increase, divide the increase 26,782 by 41,514, which is the number that increased. This gives a decimal which will be expressed as a per cent. The division is shown at the right. To get the per cent to the nearest tenth, the division is carried out to four decimal places and the result is rounded off to 0.645. Therefore, the increase in population during the 10 years was 64.5%. Ans.

```
                  .6451
41514 ) 26782.0000
        249084
         187360
         166056
          213040
          207570
            54700
            41514
```

EXAMPLE 6. The weight of material at the beginning of a purifying treatment is 5680 lb, and the weight of material after the treatment is 5400 lb. What per cent of the material was removed? Express the result to the nearest hundredth.

SOLUTION. The weight of material removed is the difference between 5680 and 5400. It is 280 lb. To find the required per cent, first divide 280 by 5680 to get a decimal, and then change this decimal to per cent. The decimal is 0.04929, so the per cent of material removed is 4.93%. Ans.

*c*) FINDING THE BASE WHEN PERCENTAGE AND PER CENT ARE KNOWN. Sometimes a number that is given is known to be a certain per cent of another number that is to be found. The given number will be the percentage. To find the number or the base when the percentage and per cent are known, divide the percentage by the per cent expressed as a decimal. This rule is based upon the following reasoning: Let's assume that 20% of a certain number is 35 and you are to find the number. Since 20% of the number is 35, you find 1% of it by dividing 35 by 20. (This is exactly the same thing as saying, "If 3 doz bananas cost 75 cents, 1 doz will cost 75¢ divided by 3, or 25¢.") Since these are 100 per cents in a whole thing or a unit, you multiply the value of 1% by 100 to find the value of the desired number. Thus $\frac{35}{20} \times 100 = 175$. This result is the same as if you had divided 35 by 20% expressed as a decimal. That is, $35 \div 0.20 = 175$. Here are some examples to show you the procedure. Use whichever reasoning seems to be more logical to you.

EXAMPLE 1. If 68% of a certain number is 101, what is the number?

SOLUTION. According to the rule, 68% is first changed to a decimal, thus becoming 0.68. Then, $101 \div 0.68 = 148\frac{9}{17}$. Therefore, the number is $148\frac{9}{17}$. Ans.

Or, using the other method: 1% of the required number is $\dfrac{101}{68}$ and 100% of the required number is $\dfrac{101}{68} \times 100 = 148\frac{9}{17}$. Ans.

**EXAMPLE 2.** If the percentage of a certain number is 784, at $83\frac{1}{3}\%$, what is the number?

**SOLUTION.** The per cent is first expressed as a fraction. For this purpose it is convenient to reduce $83\frac{1}{3}\%$ to an improper fraction; thus,

$$83\tfrac{1}{3}\% = \frac{83\frac{1}{3}}{100} = \frac{\frac{250}{3}}{100} = \frac{250}{3} \times \frac{1}{100} = \frac{250}{300} = \frac{5}{6}$$

According to the rule, 784 is to be divided by $\frac{5}{6}$ or $784 \div \frac{5}{6} = 784 \times \frac{6}{5} = 940.8$. The required number is therefore 940.8. Ans.

**EXAMPLE 3.** A furniture dealer pays $225 for a chair, and he wants his cost to be $62\frac{1}{2}\%$ of the price for which he sells the chair. What should the selling price be?

**SOLUTION.** In this example $225 is $62\frac{1}{2}\%$ of the required selling price, so, to find the selling price, divide $225 by $62\frac{1}{2}\%$ expressed as a decimal. Since $62\frac{1}{2}\%$ is 0.625, and since $225 \div 0.625 = 360$, the selling price should be $360. Ans.

As a check, you can multiply 360 by 0.625. Since $360 \times 0.625 = 225$, the result is correct.

## Check Your Learning

37. The symbol for per cent is ________.

(%)

38. In the example 6% of $40 = $2.40, 6 is the ________, 40 is the ________, and $2.40 is the ________.

(rate, or per cent) (base) (percentage)

39. When you multiply a number by a per cent, the result obtained is called the ________.

(percentage)

40. Per cent and percentage (do, do not) refer to the same thing.

(do not)

41. If 80 oz of salt solution has 4 oz of salt in it, there is ________ per cent of salt in it.

(5%)

42. If the yearly cost of a newspaper increased from $20 to $25, the per cent of increase was ________.

(25%)

43. If the price of 10 lb of potatoes dropped from 75¢ to 60¢, the per cent of decrease was ________.

(20%)

44. If 40% of my money is $10, I have ________.

($25)

## Practice Problems

1. In 1963 the profit made by a certain company was $240,811.32, and in 1964 the profit was $302,715.85. By what per cent was the profit in 1964 greater than the profit in 1963? Round off your result to the nearest tenth.
2. Before a new boiler was installed in a manufacturing plant, the average amount of coal used in a day was 3630 lb. With the new boiler the average amount of coal used in a day is 3400 lb. What per cent of the previous amount of coal needed was saved by putting in the new boiler? Round off your result to the nearest tenth of a per cent.
3. The number of rivets that will be needed in constructing the steel framework of a new building is 2432. But it is desirable to order an extra 5% of this number to allow for defective and spoiled rivets. How many rivets should be ordered? Round off your result to the nearest multiple of 10.
4. The cost of making a coat is $28.25. The maker desires to get a profit of about 40% of the cost when he sells it. What should the selling price be? Give the result to the nearest multiple of 50 cents.
5. If the maker decides that the cost of the coat in problem 4 is to be about 60% of the selling price, what should the selling price be? Express your answer as a multiple of 50 cents.
6. The number 418 is what per cent smaller than 516?
7. A class had 30 pupils at the beginning of this school term, but now it has 5 more pupils. What is the per cent of increase?
8. Gasoline, which had been selling at 28¢ per gallon, is now selling for 4¢ less per gallon. What is the rate of decrease?
9. By careful reorganization a company was able to reduce the cost of operation from $21,840 per month to $17,190 per month. By what per cent was the cost of operation reduced?
10. The circulation of a city library for September was 157,928 books; for October, 184,211 books. What was the per cent of gain for October over September?

# *Answers to Practice Problems*

## Article 7

1. $6:3$ or $\frac{6}{3}$
2. $7:4$ or $\frac{7}{4}$
3. $9:27$ or $\frac{9}{27}$
4. $20:30$ or $\frac{20}{30}$
5. $18:8$ or $\frac{18}{8}$
6. $14:21$ or $\frac{14}{21}$

## Article 9

1. $3:12$ or $\frac{1}{4}:1$
2. $240:30$ or $4:\frac{1}{2}$
3. $24:16$ or $2:1\frac{1}{3}$
4. $1\frac{1}{4}:48$ or $\frac{5}{64}:3$
5. $288:72$ or $2:\frac{1}{2}$
6. $32:400$ or $\frac{8}{25}:4$

## Article 11

1. *a)* 9
   *b)* $\frac{1}{4}$
   *c)* $\frac{12}{7}$
   *d)* $\frac{20}{3}$
2. *a)* 8.333
   *b)* 0.222
   *c)* 0.967
3. *a)* $5:1$
   *b)* $1:1.75$
   *c)* $2.5:1$
   *d)* $1:8$

## Article 14

1. *a)* $\frac{5}{9} = \frac{15}{27}$
   *b)* $\frac{3}{0.25} = \frac{12}{1}$
   *c)* $\frac{4.5}{6} = \frac{1.5}{2}$
2. *a)* $2:3 = 6:9$
   *b)* $17:15 = 68:60$
   *c)* $\frac{1}{2}:\frac{3}{8} = 4:3$
   *d)* $1:3 = \frac{1}{3}:1$
3. *a)* Two is to three as six is to nine.
   *b)* Seventeen is to fifteen as sixty-eight is to sixty.
   *c)* One-half is to three-eighths as four is to three.
   *d)* One is to three as one-third is to one.

## Article 19

1. $5:4 = 75:x$ or $\frac{5}{4} = \frac{75}{x}$
2. $25:48 = 650:x$ or $\frac{25}{48} = \frac{650}{x}$
3. $24:52 = 39:x$ or $\frac{24}{52} = \frac{39}{x}$
4. $240:200 = 8:x$ or $\frac{240}{200} = \frac{8}{x}$

## Article 21

1. 60°
2. $1248
3. $84\frac{1}{2}$ min
4. $6\frac{2}{3}$ days

## Review Problems

1. 8 hr
2. $11.25
3. 36 days
4. $87\frac{1}{2}$ min
5. $9\frac{3}{5}$ days
6. 9 hr
7. $6\frac{2}{3}$ days
8. 720 gpm
9. $10.97
10. 1000 lb

## Article 22

1. 195 lb
2. 7.74 ohms
3. 819 lb
4. $A:B = a^2:b^2$

## Article 24

1. 32 hr
2. $32\frac{1}{7}$ days
3. $510.30
4. 420 gpm
5. 3968 revolutions
6. 42 miles
7. $985.80
8. 770 yd
9. $10\frac{2}{5}$ days

## Article 28

1. 25.7%
2. 6.3%
3. 2550 rivets
4. $39.50
5. $47.00
6. 19%
7. $16\frac{2}{3}$%
8. 14.3%
9. 21.3%
10. 16.6%

5/30/.6   $\frac{3}{4} \times \frac{3}{2}$   $\frac{9}{8}$/9/1.125

$$\begin{array}{r} .033 \\ 750\overline{)2500} \\ 2250 \\ \hline 2500 \\ 2250 \end{array}$$

$$\begin{array}{r} 8 \\ \hline 10 \\ 8 \\ \hline 20 \\ 18 \\ \hline 40 \end{array}$$

$$\frac{\cancel{11}}{\cancel{8}} \times \frac{\cancel{16}^{2}}{\cancel{11}}$$

# Practical Arithmetic

Serial 6750F　　PART 6　　Edition 1

## *Examination Questions*

**Notice to Students.—Study this instruction text thoroughly before you answer the following questions. Read each question carefully and be sure you understand it. Show enough of your work to indicate how your answers were obtained. We will not accept answers alone. When you complete your work, examine it closely, correct all the errors you can find, and see that every question is answered; then mail your work to us. DO NOT HOLD IT until another examination is ready.**

The point value of each question is in parenthesis after the number of the question.

1. (4) Change the following per cents to equivalent fractions and then reduce the fractions to lowest terms.

*a*) $8\frac{1}{3}\%$　*b*) $3\frac{1}{8}\%$　*c*) $52\frac{1}{2}\%$　*d*) $\frac{2}{7}\%$

2. (4) Write the following numbers as per cents:

*a*) 0.75 75%　*b*) 0.032 3.2%　*c*) 1.5 150%　*d*) $\frac{3}{8}$ 37.5%

3. (4) Write the following per cents as decimals:

*a*) $12\frac{1}{2}\%$ .125　*b*) 1.75% 0.0175　*c*) $\frac{1}{2}\%$ .5　*d*) 0.012% .00012

4. (8) Rewrite each of the following ratios so that the smaller term is 1. If the larger term is not a whole number, express it as a mixed number.

*a*) 30:5 6:1　*b*) 3:7 $1:2\frac{1}{3}$　*c*) 5:2 $2\frac{1}{2}:1$　*d*) $\frac{1}{4}:\frac{2}{3}$ $1:2\frac{2}{3}$

5. (8) What is the value of each of the following ratios?

*a*) 3:5 0.6　*b*) $\frac{3}{4}:\frac{2}{3}$ 1.125　*c*) 2.5:75 .033　*d*) $1\frac{3}{8}:\frac{11}{16}$ 2.

**Complete work is required in Questions 6, 7 and 8**

6. (8) A certain kind of brass is 63% copper and 37% zinc. How many pounds of copper and how many pounds of zinc are there in 324 lb of this brass? Give your answers to the nearest tenth of a pound.

7. (8) A ship has traveled 1564 miles. If this is 85% of its voyage, how much farther does it have to go?

8. (8) *a*) The number 35 is what per cent of the number 20? 175

*b*) The number 28 is 12% of what number? $233\frac{1}{3}$

9. (8) A term has been omitted in each of the following proportions. What is the value of the missing term?

*a*) $2:3 = \underline{4}:6$

*b*) $3:5 = 9:\underline{15}$

*c*) $\underline{7}:21 = 1:3$

*d*) $3:\underline{4} = \frac{1}{2}:\frac{2}{3}$

**Solve problems 10 to 13 by proportion. Be sure to write the proportion that you use and show how you solved it.**

10. (8) A garrison of 600 men has provisions sufficient to last for 15 days. If a reinforcement of 150 men arrives, how long will the provisions last?

$750:600 = 15:x$

$x$ = provisions

11. (8) If 12 men require 30 days to perform a certain amount of work, how long will 8 men require for the same work?

$12:8 = x:30$

12. (8) The relative speeds of two trains are as 4:3. How many hours will be required by the slower train to travel the same distance covered by the other train in 8 hr?

$4:3 = 8:x$

13. (8) The wind pressure on a flat surface is proportional to the square of the speed of the wind. The pressure on a certain surface is 120 lb when the wind is blowing at 20 miles an hour. What will be the pressure on the surface when the wind is blowing at 40 miles an hour?

$20^2:40^2 = 120:x$

**Solve Question 14 by the Unit Method.**

14. (8) If 3 men can build 18 cabinets in 2 days, how many days will it take 5 men to build 60 cabinets?

$$\frac{3 \times 2 \times 60}{18 \times 5}$$

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